Identification and Optimization of Aircraft Dynamics

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A technique is described for the design of an adaptive controller for multivariable systems and is based on recently developed methods for identification and optimization. An application of the method to a helicopter system with time-varying parameters is considered in detail. The response of the adaptive system is compared with the corresponding response of a system with a fixed controller and a system using optimal control. The comparison reveals the almost optimal character of the adaptive system.

Nomenclature

A	= $n \times n$, system matrix
B	= $n \times m$, input matrix
C	= $n \times n$, model matrix
F	= $m \times n$, feedback matrix
G	= $n \times n$, model matrix (estimate of A)
H	= $n \times m$, model matrix (estimate of B)
K	= $n \times n$, symmetric Riccati matrix
P	= $n \times n$, symmetric positive definite matrix used in the model
Q, S	= $n \times n$, symmetric positive semidefinite matrices of the performance index
Q_1	= $n \times n$, symmetric positive definite Lyapunov matrix
R	= $m \times m$, symmetric positive definite matrix of the performance index
T	= superscript denoting the transpose of a vector or a matrix
$a_{ij}, b_{ij}, c_{ij}, g_{ij}, h_{ij}, p_{ij}$	= i, jth element of the matrix denoted by the respective upper case letter
e	= n-dimensional state error vector
m	= integer denoting the number of inputs
n	= integer denoting the number of aircraft states
r	= command input vector (m-dimensional)
t	= time
u	= the m-dimensional open loop input vector
x	= aircraft state vector (n-dimensional)
y	= the n-dimensional model state vector (estimate of x)
Γ	= $n \times n$ coefficient matrix of the identification model
Λ	= $n \times m$ coefficient matrix of the identification model
$\gamma_{ij}, \lambda_{ij}$	= i, jth element of the corresponding matrix Γ or Λ
$(\dot{\ })$	= time derivative of the corresponding entity
$*$	= denotes optimality

I. Introduction

AN adaptive system is defined as one which uses the information generated while it is in operation in contrast to conventional control systems in which a priori information is used. As a consequence, adaptive systems are expected to perform satisfactorily over a wider range of variations of the system parameters and the environment. Over the past decades various techniques have been suggested for the synthesis of adaptive systems, but very few of them have found practical application in complex control situations. In this paper, an adaptive technique for multivariable systems is suggested that uses recently developed methods for identification and optimization. The feasibility of this technique in a practical context is demonstrated by considering in detail its application to the adaptive control of helicopter dynamics.

While designing a controller for an aircraft system, the dynamics of the system are generally linearized around a nominal air speed. Parameters of the system such as gross weight are assumed to be constant and the structure of the aircraft is also considered to be fixed. This results in a set of linear time-invariant differential equations and the designer generally computes a set of feedback gains using a quadratic performance criterion[1] to obtain satisfactory performance of the aircraft. Using such an approach controllers have been developed for VTOL systems and have proved adequate when the system parameters do not vary over a wide range. However, in actual operation, the gross weight, the air speed, the location of the center of gravity and the altitude of VTOL systems vary with time, resulting in substantial changes in their dynamics. A fixed controller is found to be inadequate to achieve satisfactory performance during these widely different flight conditions and an adaptive controller becomes almost mandatory.

The identification procedure suggested in Ref. 2 is used here to continuously track the parameters of the system while the feedback parameters of the controller are updated using an algorithm suggested in Ref. 3. The identification scheme is discussed briefly in Sec. III and the iterative procedure for adjusting feedback parameters is considered in Sec. IV. The identification scheme uses a mathematical model whose parameters are adjusted using the inputs $u(t)$, the state variables $x(t)$ of the aircraft and the error $e(t)$ between the state vector of the model and the aircraft. The controller equations are derived so that a quadratic Lyapunov function assures the stability of the identification scheme. In the iterative procedure used for computing the feedback parameters a quadratic algebraic equation in a matrix K is solved by iteratively solving a set of linear equations.

Computer simluation reveals that this approach of identification followed by optimization is a feasible one and results in satisfactory performance even when there are wide variations in the air speed resulting in significant fluctuations in the system parameter values.

The procedure presented in this paper cannot be directly implemented on most VTOL systems at the present time since onboard digital computers are as yet not very

Received August 8, 1972; revision received November 13, 1972. The first author would like to thank J. Maciolek, S. O'Connor and D. Fowler of United Aircraft Corp., Sikorsky Aircraft Division for many stimulating discussions. The research reported in this paper was sponsored in part by support extended to Yale University by the U.S. Office of Naval Research under contract N0014-67-A-0097-0020-NR375-131.
Index category: VTOL Handling, Stability, and Control.
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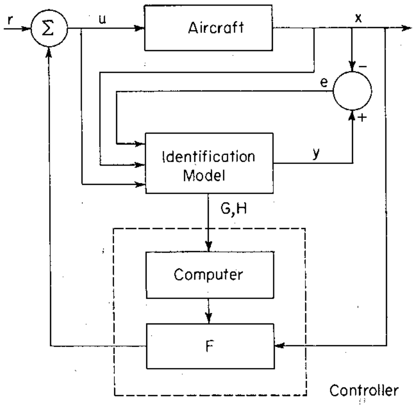

Fig. 1 The adaptive control model.

common. However, it is felt that in the near future, as the requirements on the performance of such systems become more stringent, adaptive procedures using onboard computers will become the rule rather than the exception.

II. Statement of the Problem

A dynamical system is described by the vector differential equation

$$\dot{x} = [A(t) + B(t)F(t)]x + B(t)r(t) \quad (1)$$

where x is an n-dimensional state vector and $r(t)$ an m dimensional input vector, and the system matrices $A(t)$ and $B(t)$ are (nxn) and (nxm) respectively. $F(t)$ is an (mxn) matrix of feedback gains. If $A(t)$ and $B(t)$ are time-varying and the terminal time T is finite, the feedback matrix $F(t)$ can be chosen to minimize a quadratic performance index of the form

$$\frac{1}{2}x^T(T)Sx(T) + \int_{t_0}^{T} [x^T(t)Qx(t) + r^T(t)Rr(t)]dt$$

where the matrices S and Q are positive semidefinite and the matrix R is positive definite. The solution to this problem is well known[4] and is obtained by integrating a matrix Riccati differential equation

$$\dot{K}_1 = A^TK_1 + K_1A - K_1BR^{-1}B^TK_1 + Q; \; K_1(T) = S \quad (2)$$

The time-varying feedback matrix $F_1(t)$ is then given by

$$F_1(t) = -R^{-1}B^T{}_{K_1(t)} \quad (3)$$

and constitutes the complete solution to the problem. For the particular case when $S = 0$ and Q and R are time-invariant and $T \rightarrow \infty$ (infinite time problem) the optimal feedback matrix $F_1(t)$ tends to be a time-invariant matrix given by

$$F_1 = -R^{-1}B^T{}_{K_1} \quad (4a)$$

where K_1 (nxn symmetric matrix) satisfies the algebraic Riccati equation

$$A^TK_1 + K_1A - K_1BR^{-1}B^TK_1 + Q = 0 \quad (4b)$$

In the adaptive problem that we shall consider, some of the elements of the matrices $A(t)$ and $B(t)$ vary with time in an unknown fashion but within specified bounds. The design problem is to compute and implement on-line, a feedback matrix $F(t)$ which would result in satisfactory response throughout the entire flight regime of the aircraft. In Sec. VI this design procedure is carried out in two stages (Fig. 1): i) estimation of the time-varying elements of the A and B matrices, and ii) determination of the feedback matrix using the estimates of A and B.

The concept of identifying a system prior to optimizing it is not by any means a new one; most of the early literature in the area of adaptive control is concerned with such an approach. However, until recently, satisfactory methods for the rapid identification of a multivariable system were not available and the problem of solving the matrix Riccati equation on-line also posed difficult problems. It is the proper combination of recent developments in these two areas that has resulted in the adaptive procedure presented in this paper. The speed and accuracy of the identification procedure and the convergence properties of the iterative scheme for computing the feedback matrix naturally play a crucial role in the success of the overall procedure. In the adaptive control problem of a helicopter, treated in detail in Sec. VI, the schemes suggested are found to have the requisite properties and the overall system is found to be stable.

III. The Identification Scheme

Let a dynamical system be described by the n-dimensional vector differential equation

$$\dot{x} = Ax + Bu(t) \quad (5)$$

where A and B are constant matrices with unknown elements. To estimate the parameters of A and B a model is set up described by the equation

$$\dot{y} = Cy + [G(t) - C]x + H(t)u(t) \quad (6)$$

where y is an n-vector, C is a stable matrix and the matrices $G(t)$ and $H(t)$ have time-varying elements. Using Lyapunov's direct method it was shown[2] that the elements of $G(t)$ and $H(t)$ can be adjusted continuously so that they estimate the corresponding elements of A and B asymptotically. Since C is a stable matrix, a positive definite matrix P exists[5] such that the equation

$$C^TP + PC = -Q_1 \quad (7)$$

is satisfied, where Q_1 is a positive definite matrix. The

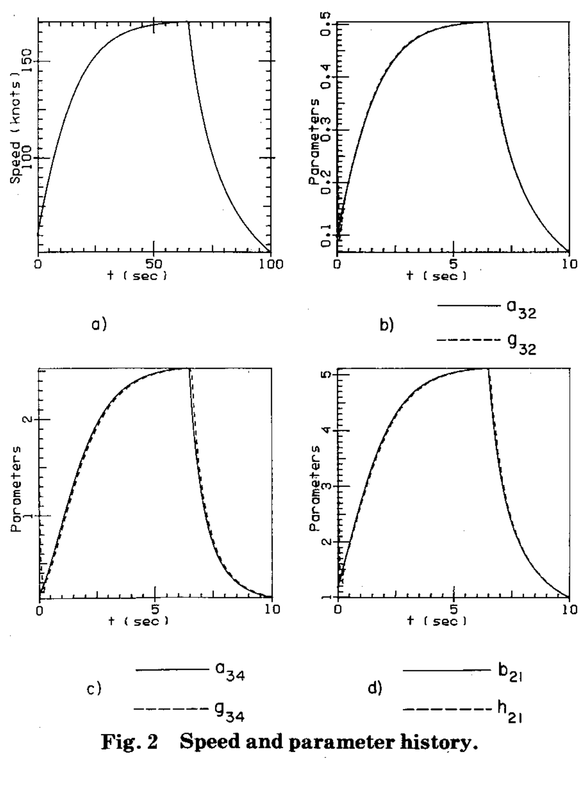

Fig. 2 Speed and parameter history.

control laws which govern the changes in the matrices $G(t)$ and $H(t)$ are then given by

$$\dot{G}(t) = -\Gamma \otimes (P\ \mathrm{e}x^T)$$
$$\dot{H}(t) = -\Lambda \otimes (P\ \mathrm{e}u^T) \tag{8}$$

where Γ and Λ are matrices with positive elements, e is the error vector $y - x$, and $\otimes$ denotes element by element multiplication of two matrices. This procedure leads to asymptotic identification of A and B, i.e., $\lim_{t\to\infty} G(t) = A$; $\lim_{t\to\infty} H(t) = B$ if $u(t)$ is a general input. Further, if the matrices A and B vary slowly with time it can also be demonstrated that the same scheme results in a continuous tracking of the elements of A and B with a bounded error.

The only constraints imposed on the matrices C, Γ, Λ and Q_1 so far are that C be a stable matrix, Γ and Λ have only positive elements and Q_1 be positive definite. As might be expected, the speed of response of the identification scheme depends very much upon the choice of these matrices; this is considered briefly in Sec. VI.

IV. Solution of the Algebraic Riccati Equation (4b)

When the system matrices A and B are constant and known, and the matrices Q and R in the performance index are specified the solution of the infinite time problem was given in terms of the gain matrix F_1 satisfying the relation (4a), where K_1 satisfies the algebraic equation (4b). In practice K_1 is obtained as the asymptotic solution of the Riccati equation (2).

In the adaptive control situation, the identification procedure yields at any instant only the estimates G and H of the system matrices A and B. Considering the frozen system defined by the matrices G and H, a corresponding optimal feedback matrix F may be defined as

$$F = -R^{-1}B^TK \tag{9a}$$

where the matrix K satisfies the algebraic equation

$$G^TK + KG - KHR^{-1}H^TK + Q = 0 \tag{9b}$$

If $\lim_{t\to\infty} G = A$ and $\lim_{t\to\infty} H = B$, the matrix K will tend to K_1 and the feedback matrix F in turn will tend towards the true optimal matrix F_1. However since the estimates G and H are time-varying the matrix K satisfying Eq. (9b) cannot be determined asymptotically as in the time-invariant case. Further, this equation is quadratic in the matrix K so that numerical procedures are required for its solution.

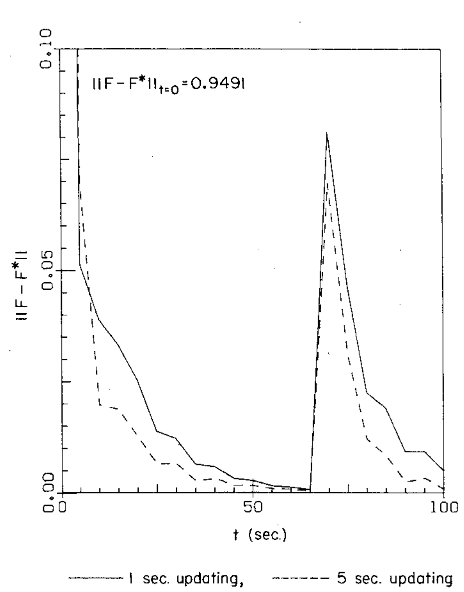

Fig. 3 Deviation of F from F†.

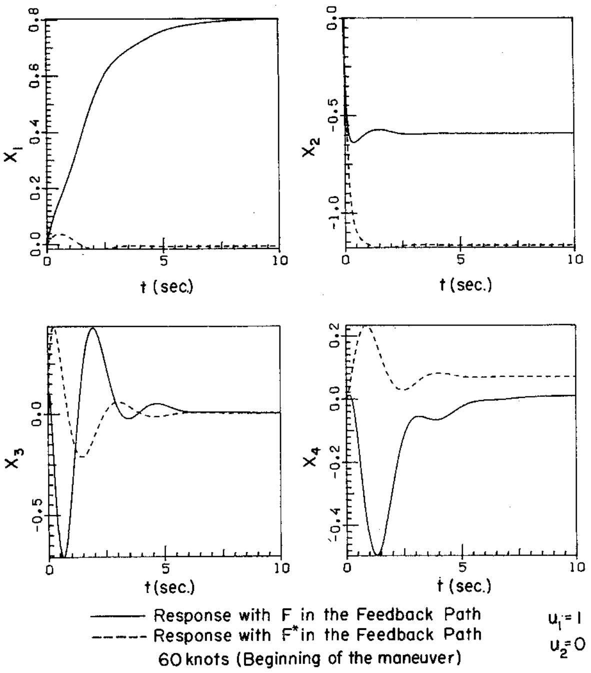

Fig. 4 Step response.

The procedure suggested in Ref. 3 represents an iterative method for the solution of Eq. (9b). The iterative algorithm involves the following steps: a) Choose a matrix F^0 such that the matrix G^0 given by $G^0 = G + HF^0$ is stable. b) Solve for K^{i+1} the linear equation

$$G^{i^T}K^{i+1} + K^{i+1}G^i + Q + F^{i^T}RF^i = 0$$

Compute

$$F^{i+1} = -R^{-1}H^TK^{i+1} \tag{10}$$

and

$$G^{i+1} = G + HF^{i+1}$$

As $i \to \infty$ the above scheme yields the solution of the algebraic equation (9b). In the adaptive scheme suggested in this paper only one iteration of the above algorithm is carried out at any instant of time. This implies that only one matrix inversion is needed in Eq 10 at every instant of time the feedback matrix is updated. For high dimensional systems such a procedure makes real time processing of data possible using small onboard computers.

As the system varies slowly with time the identification scheme continuously tracks the system parameters. The estimates G and H are then used to update the feedback matrix periodically, using the iterative scheme (10). It is ooserved that for the success of the adaptive scheme, the identification model must track the system accurately so that the optimal feedback matrix computed for the model is also close to the optimal feedback matrix of the system.

V. Choice of Parameters of the Performance Index

Before proceeding to apply the identification and optimization procedures discussed so far to practical problems, some questions regarding the choice of certain matrices and parameters of the overall system must be resolved.

Q and R Matrices

Even for optimal control problems where the system parameters are completely known, the choice of the Q and

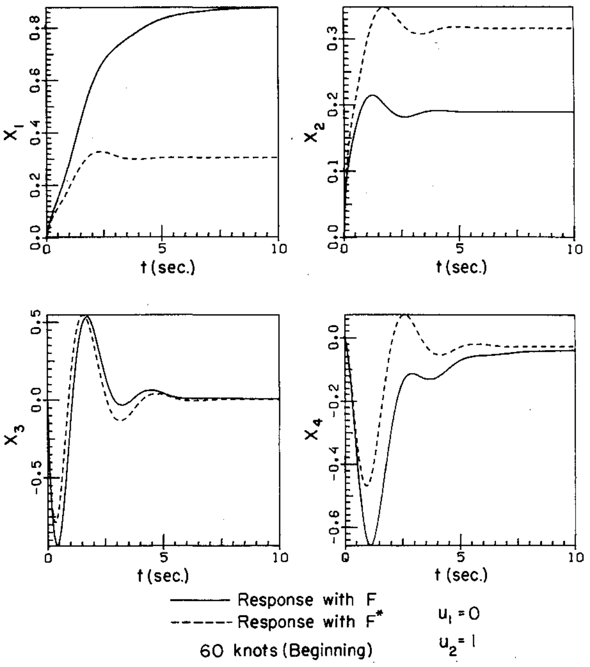

Fig. 5 Step response.

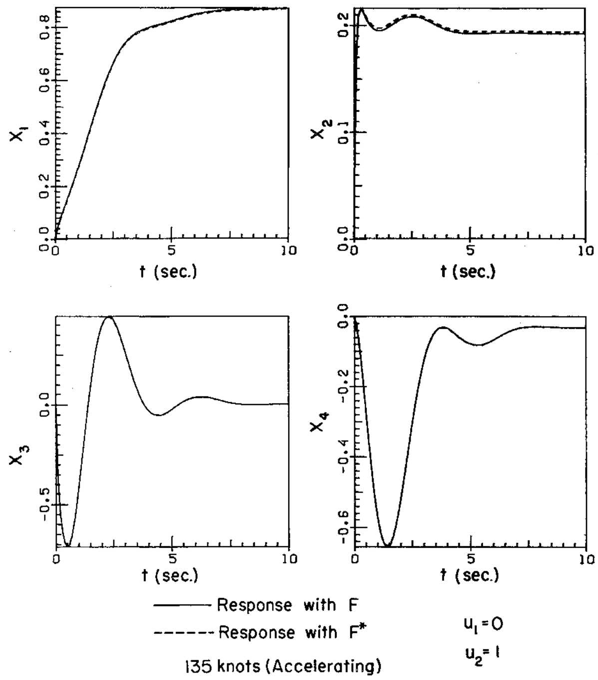

Fig. 7 Step response.

R matrices in the performance index is not an easy one. For example, in aircraft systems these matrices have to be related to the subjective criterion of the pilot and methods for choosing the elements of these matrices are discussed in Ref. 1. A commonly followed procedure is to make Q and R diagonal with the elements of the matrices inversely proportional to the square of the maximum allowable variations of the state variables and control variables respectively (i.e., $q_{ii} = 1/2\bar{x}_i$, $r_{ii} = 1/2\bar{v}_i$ where $\bar{x}_i$ and $\bar{v}_i$ are the bounds on the state variable x_i and the control variable v_i).

While the above procedure is satisfactory for a time-invariant plant, it is generally inadequate for a time-varying one. Since by definition, in an adaptive system the plant parameters vary with time, the performance index (i.e., Q and R matrices) should also in general vary with the changing plant characteristics. However, except in very special cases, choosing Q and R as functions of environmental parameters is a very difficult problem. Hence the adaptive scheme suggested here is applicable mainly to those systems where a single performance index is found to be adequate for all parameter variations of the system. From experience it is found that this is a satisfactory assumption to make with many helicopter systems in forward flight (this is not true however if hover conditions are included). Hence in the example discussed Q and R matrices are assumed to be constant.

VI. Example: Adaptive Control of a Helicopter

We consider in this section, in some detail, the feasibility of applying the design procedure developed earlier to the practical problem of synthesizing a controller for a helicopter operating in widely differing flight regimes. While

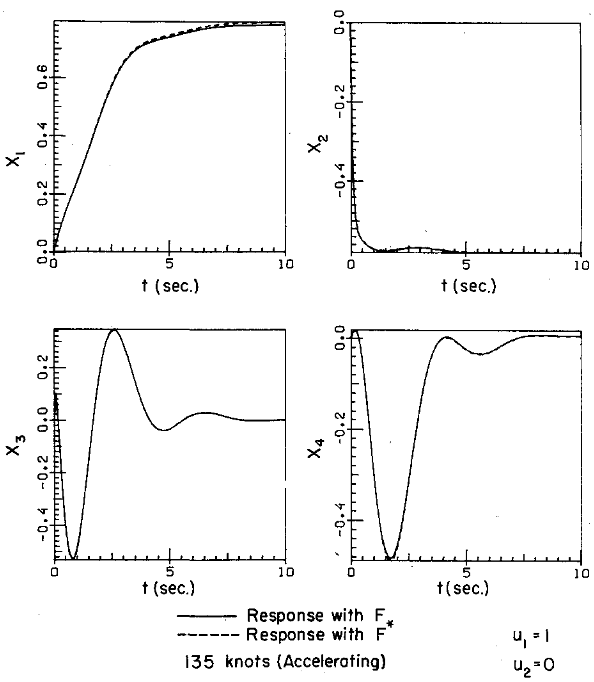

Fig. 6 Step response.

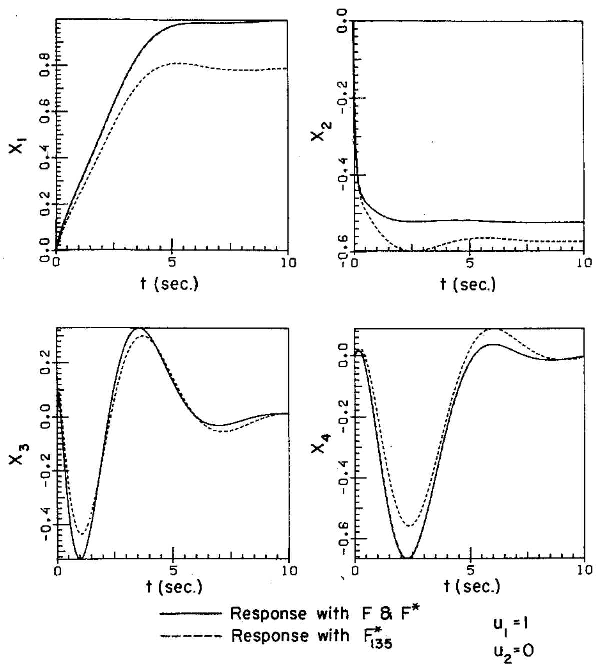

Fig. 8 Step response (170 knots).

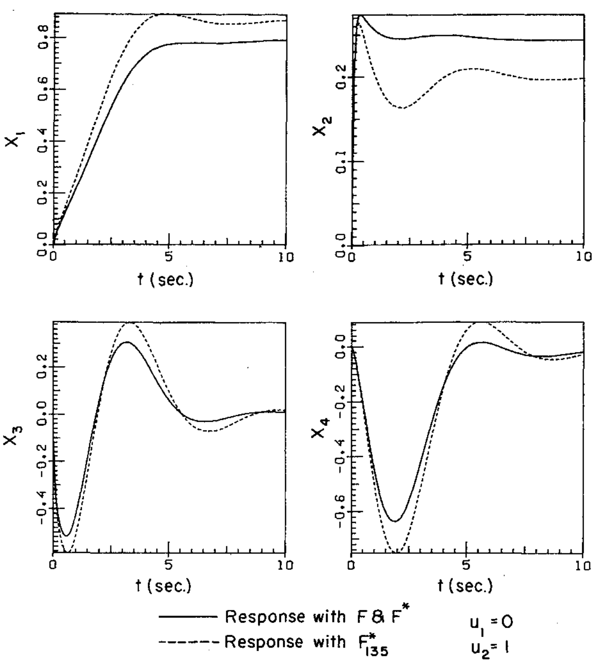

Fig. 9 Step response (170 knots).

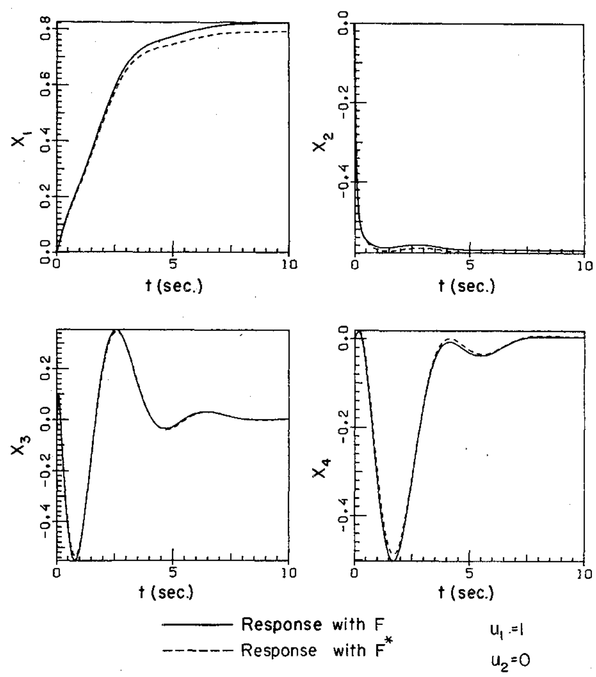

Fig. 10 Step response (135 knots, decelerating).

the procedure can be applied to any situation in which the dynamics of the system vary with time, we consider here only those variations produced by changes in the airspeed in the longitudinal motion of the aircraft.

Aircraft Dynamics

The dynamics of a helicopter in the vertical plane is described by the vector differential equation

$$\dot{x} = Ax + Bu$$

where A is a (4x4) matrix, B is a (4x2) matrix, x is a 4-dimensional vector and u, a 2-dimensional vector. The state variables are: x_1—horizontal velocity; x_2—vertical velocity; x_3—pitch rate; and x_4—pitch angle. The control inputs are given by u_1—collective and u_2—longitudinal cyclic. For typical loading and flight conditions of the helicopter at an airspeed of 135 knots, the matrices A and B are:

$$A = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.420 \\ 0 & 0 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{bmatrix}$$

As the airspeed changes all the elements of the first three rows of both matrices also change. The most significant changes take place in the elements a_{32}, a_{34} and b_{21} and in the following discussion all the other elements are assumed to be constant. Figure 2 indicates how these parameters vary as a function of time; the range of airspeed is from 60 knots to 170 knots. The bounds on the parameter values are given by $0.06635 < a_{32}(t) < 0.5047$; $0.1198 < a_{34}(t) < 2.526$; $0.9775 < b_{21}(t) < 5.112$.

The Environment

The time-varying environment in this case is provided by the varying air speed which results in time-varying parameters. For simulation purposes, the parameters were varied according to a preprogrammed velocity vs time function Fig. 2a and not as a function of the state x_1 which represents the deviation of the horizontal velocity from trim condition. The velocity curve in Fig. 2a indicates a rapid acceleration from 60 knots and rapid deceleration from 170 knots, reflecting the availability of greater power for these maneuvers at these air speeds. The problem, under these conditions, is to design a controller so that the over-all dynamics of the system is satisfactory at all air speeds.

Fixed Controller Gains

A relatively simple solution to the problem is the use of a fixed set of gains in the feedback path. Fixed gains which are almost always preferred in aircraft systems, are usually justifiable on the basis of simplicity and cost of implementation. The adaptive controller suggested in this paper is considerably more complex and hence more expensive to implement. Prior to using such an adaptive controller in the aircraft system it is necessary to decide whether the increased cost and complexity of the controller are justified by the improvement in performance.

The fixed feedback gains used for comparison purposes were calculated for an airspeed of 135 knots, and were obtained by minimizing a quadratic performance index

$$\int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt$$

where $Q_{11} = 0.04$, $Q_{22} = 0.25$ and all other elements of the Q matrix are zero and R is a diagonal matrix with $R_{11} = 1/25$ and $R_{22} = 1/9$.

The use of this feedback gain matrix at an air speed of 60 knots resulted in underdamped response and in an overdamped response at 170 knots. The performance of the aircraft with a fixed controller, deteriorates as the airspeed deviates from the nominal value of 135 knots. Many present day helicopters with fixed feedback gains exhibit such changes in handling qualities but are still deemed

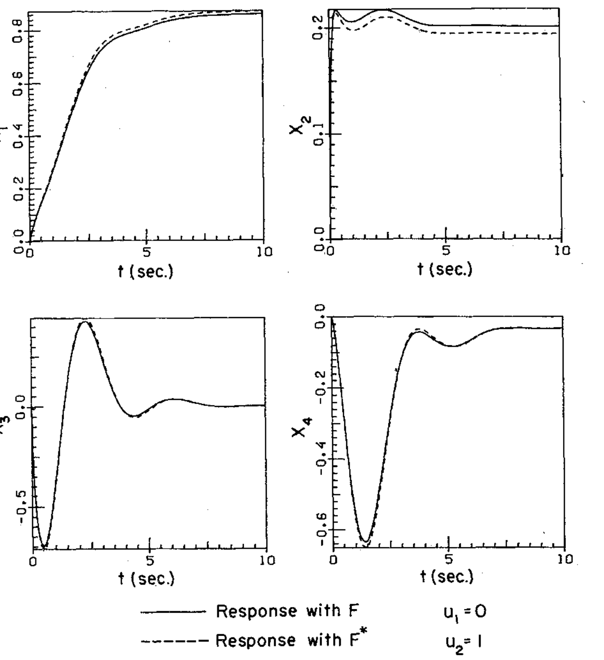

Fig. 11 Step response (135 knots, decelerating).

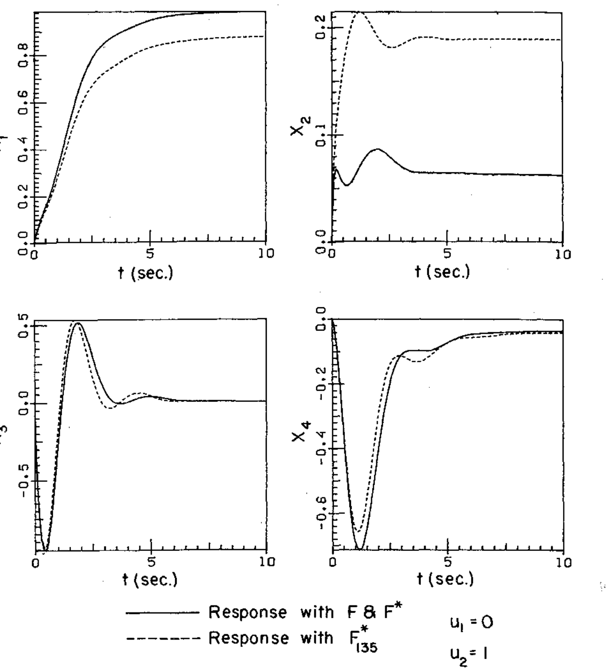

Fig. 13 Step response (60 knots, decelerating).

adequate. It is obvious however that as the range of operation of the helicopter increases (e.g., 30 to 210 knots) a stage will be reached when the deterioration in the performance of the system becomes intolerable.

The Identification Scheme

It was mentioned in Sec. III that when the inputs are sufficiently general, the identification scheme suggested yields asymptotic stability so that output errors as well as parameter errors tend to zero. For the optimization procedure to be satisfactory, the tracking of the system by the model must be efficient. In the aircraft system under consideration significant changes in the parameters of the A and B matrices take place in about 10 sec (by significant change we mean a change which results in an observable change in the response of the system with a fixed matrix of feedback gains). The model tracks the system with approximately a time constant of 1 sec, so that the identification scheme is found to be more than adequate for the variations of parameters encountered in practice.

The parameters of the identification scheme include the elements of the matrices C, P, Γ and Λ. The matrix C was chosen to be a stable diagonal matrix. Since only the second and third rows of the A and B matrices are to be identified it is necessary only to choose the two elements C_{22} and C_{33}. The choice of a diagonal matrix C and a diagonal Q_1 makes the matrix P also diagonal. The only constraint on the elements of P is that they be positive. The elements of C and P were chosen to be

$$C_{22} = -2.5; \quad C_{33} = -5.0; \quad P_{11} = 30;$$
$$P_{22} = 25; \quad P_{33} = 25; \quad P_{44} = 30$$

The three parameters of the matrices Γ and Λ were chosen for rapid identification and satisfactory values were found to be

$$\gamma_{32} = 2.44; \quad \gamma_{34} = 60.00; \quad \lambda_{21} = 5.0$$

The resulting identification model as mentioned earlier tracked the parameter changes in the system with a time constant of 1 sec. It is worth mentioning that due to the number of parameters that can be adjusted, considerable freedom exists in the choice of the values of these parameters. Consequently, trial and error methods in their choice are unavoidable at this stage.

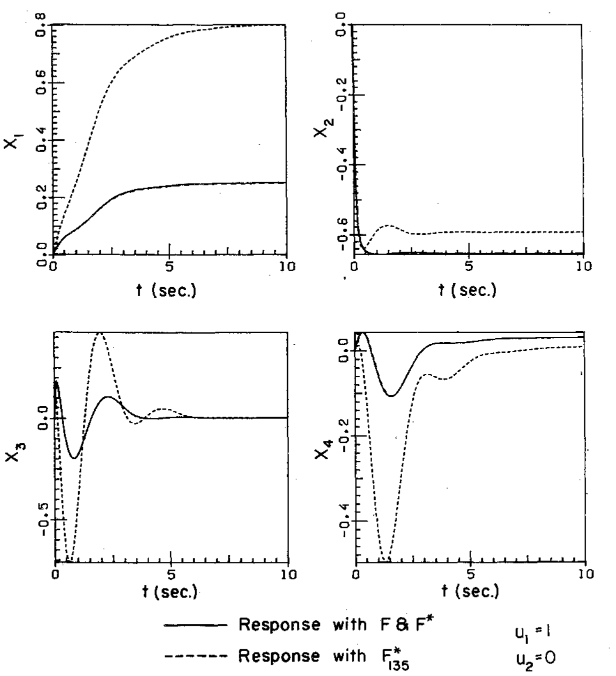

Fig. 12 Step response (60 knots, decelerating).

The Adaptive Scheme

The simulation of the entire adaptive scheme was carried out on a digital computer. For the integration of the differential equations a Δt of 0.01 sec was used. Larger values of Δt were found to lead to numerical instability. The feedback matrix was updated every second and resulted in slow variation of the elements of the feedback matrix. This fact makes it possible to use a small digital computer on board the aircraft to carry out the matrix inversion. For higher dimensional system, updating can be done only at larger time intervals.

Evaluation of the Adaptive Scheme

The adaptive scheme suggested can be evaluated using a number of arbitrary criteria. For the aircraft system the evaluation was based on: i) comparison of the response of the adaptive system with the response of the system with a fixed controller (i.e., F^*_{135}); ii) comparison of the adaptive system with the optimal system when the time-variations in the system are known; and iii) computation of error between adaptive control parameters and the corresponding optimal control parameters.

The parameters of the system were varied in a preprogrammed manner as described earlier (Fig. 2a) and the identification and optimization schemes were used as described. Figure 2(b–d) indicate how the three system parameters and their corresponding estimates as determined by the identification scheme, vary with time. Figure 3 indicates the norm of the error between the optimal controller parameters and adaptive controller parameters i.e., $\|F - F^*\|$, as a function of time. This norm is seen to decrease monotonically as the system acceleration decreases and displays a temporary increase in magnitude when the deceleration of the system is large. Another interesting feature of the observed response is that when the feedback controller is updated every 5 sec rather than every second the norm $\|F - F^*\|$ is found to be smaller during most of the interval.

In the simulations, the air speed changes from 60 knots to 170 knots and decreases back to about 51 knots. At five air speeds 60, 135, 170, 135 and 60 knots the step responses of the system due to a unit step in $u_1(t)$ and $u_2(t)$ are plotted i) with the optimal controller, ii) with a fixed controller corresponding to the optimal controller at 135 knots and, iii) with the adaptive controller in Figs. 4–13. The improvement due to the adaptive controller at 60 and 170 knots is obvious and at 135 knots the three responses are found to be approximately the same.

The rates of change of the system parameters considered in the simulations are quite reasonable ones and are those that are encountered in helicopters in operation at the present time. The identification and optimization procedure described can be used successfully for significantly more rapid changes, so that its obsolescence is not imminent.

Throughout the paper it has been assumed that no observation noise is present in the system. The presence of such noise does very little to alter the results presented in this paper when the noise level is low. Such was indeed the case in the example of the aircraft system described in this section. For higher noise levels the identification procedure yields bounded parameter errors and the feasibility of the approach depends on the sensitivity of the optimization procedure to such errors. In some cases filtering of system outputs may be necessary before applying the procedure outlined.

VII. Conclusion

An adaptive procedure is described in the paper for the control of a multivariable system with time-varying parameters. The procedure involves the identification of the time-varying system and the updating of a feedback matrix by solving an algebraic Riccati equation.

The application of the method to a helicopter system indicates that for the parameter variations encountered in practice, the method is eminently suitable and vields an overall system response which is close to the optimal response. The procedure however requires an onboard computer which at present is not available in many VTOL systems.

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